SOLVE $x\left(x\frac{dy}{dx}+2y\right)=(1)\,x$

NOTE: $2 \neq (x)'$

$$x^2\frac{dy}{dx}+2xy=x$$

NOTE: $2x=(x^2)'$

$\vdots$

HOW TO FIND ?

THIS INTEGRATING FACTOR – A FACTOR YOU MULTIPLY THE DE BY TO MAKE IT EASIER TO SOLVE

(LINEAR 1ST ORDER ODE)

GIVEN $a_1(x)\frac{dy}{dx}+a_0(x)y=b(x)$

$$\frac{dy}{dx}+\frac{a_0(x)}{a_1(x)}y=\frac{b(x)}{a_1(x)}$$

$$\frac{dy}{dx}+p(x)y=g(x)$$

$$\frac{dy}{dx} + p(x)y = g(x)$$

SUPPOSE $\mu(x)$ IS AN INTEGRATING FACTOR

$$\mu(x)\frac{dy}{dx} + \mu(x)p(x)y = \mu(x)g(x)$$

WE WANT $\mu(x)p(x) = \frac{d}{dx}\mu(x)$ (NEED TO FIND $\mu(x)$)

$$\mu p = \frac{d\mu}{dx}$$

$$\int p \, dx = \int \frac{1}{\mu} \, d\mu$$

$$\ln|\mu| = \int p(x)\,dx + C$$

$$|\mu| = Ce^{\int p(x)dx}$$

$$\boxed{\mu = Ce^{\int p(x)dx}}$$

TYPICALLY USE $C=1$

$$\mu = e^{\int p(x)dx}$$

THEN MULTIPLY μ AGAINST

CONSIDER $x\frac{dy}{dx} + 2y = 1$

(ALREADY KNOW I.F. $= x$)

$$\frac{dy}{dx} + \underbrace{\boxed{\frac{2}{x}}}_{p(x)} y = \frac{1}{x}$$

$$\mu = e^{\int p(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = e^{\ln|x|^2} = |x|^2 = x^2$$

$$\underline{x^2}\frac{dy}{dx} + \underline{2x}y = x$$

MANDATORY CHECKPOINT: $(x^2)' = 2x$ ✓

↳ CHECK DERIV OF COEF OF y'

= COEF OF y

SOLVE $(t+y+1)dt - dy = 0$

NON-LINEAR

$$(t+y+1)\frac{dt}{dy} - 1 = 0$$

$$t+y+1-\frac{dy}{dt}=0$$

$$\frac{dy}{dt}-t-y-1=0$$

$$\frac{dy}{dt}-y=t+1$$

$$\mu = e^{\int -1\,dt} = e^{-t}$$

MANDATORY CHECKPOINT: $(e^{-t})' = -e^{-t}$ ✓

$$e^{-t}\frac{dy}{dt}-e^{-t}y=(t+1)e^{-t}$$

$$\frac{d}{dt}(e^{-t}y)=(t+1)e^{-t}$$

$$e^{-t}y=\int(t+1)e^{-t}dt$$

$$e^{-t}y=-(t+2)e^{-t}+C \qquad \cdot e^{t}$$

$$y=-t-2+Ce^{t}$$

u		dv
$t+1$	$+$	e^{-t}
1	$-$	$-e^{-t}$
0		e^{-t}

$$-e^{-t}(t+1)-e^{-t}$$

$$=-e^{-t}(t+1+1)$$

$$=-e^{-t}(t+2)$$

$$=-(t+2)e^{-t}$$

SOLVE $(1+x)y' - xy = x + x^2$

$$\boxed{y' - \frac{x}{1+x}y = x}$$

$$\boxed{\mu = e^{\int -\frac{x}{1+x}dx} = e^{\int (-1+\frac{1}{x+1})dx}}$$

$$\boxed{= e^{-x+\ln|x+1|}} = e^{-x}e^{\ln|x+1|}$$

$$= |x+1|e^{-x}$$

USE $\boxed{\mu = (x+1)e^{-x}}$

$$\boxed{(x+1)e^{-x}y' - xe^{-x}y = (x^2+x)e^{-x}}$$

$$\frac{d}{dx}\left((x+1)e^{-x}y\right) = (x^2+x)e^{-x}$$

$$(x+1)e^{-x}y = \int (x^2+x)e^{-x}dx + C$$

$$\boxed{(x+1)e^{-x}y = (-x^2-3x-3)e^{-x} + C}$$

$*\ \frac{e^x}{x+1}$

$$\boxed{y = \frac{-x^2-3x-3}{x+1} + \frac{Ce^x}{x+1}}$$

$$\begin{array}{r|l} x+1 & \overline{\ \ 1\ \ } \\ & x \\ & \underline{-x-1} \\ & -1 \end{array}$$

$1 - \frac{1}{x+1}$

MANDATORY CHECKPOINT

$$\frac{d}{dx}(x+1)e^{-x}$$

$$= 1\cdot e^{-x} + (x+1)(-e^{-x})$$

$$= (1-x-1)e^{-x}$$

$$= -xe^{-x} \checkmark$$

u		dv
x^2+x	$+$	e^{-x}
$2x+1$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0		$-e^{-x}$

$$e^{-x}(-x^2-x$$
$$-2x-1$$
$$-2)$$
$$= (-x^2-3x-3)e^{-x}$$